

AD-A239 717



2

GRADUATE AERONAUTICAL LABORATORIES
CALIFORNIA INSTITUTE OF TECHNOLOGY

FM 90-3

A Comparison of Several Theories which Use Jet Scaling
to Predict Liftoff Heights of Turbulent-Jet Flames

Jay A. Hammer

Firestone Flight Sciences Laboratory

Guggenheim Aeronautical Laboratory

Karman Laboratory of Fluid Mechanics and Jet Propulsion

91-09049



Pasadena

91 8 27 080

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION Unclassified			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S) N00014-89-J-1991			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION Graduate Aeronautical Labs California Inst. of Technology		6b. OFFICE SYMBOL (If applicable)		7a. NAME OF MONITORING ORGANIZATION Office of Naval Research	
6c. ADDRESS (City, State, and ZIP Code) 1201 E. California Boulevard Pasadena, California 91125			7b. ADDRESS (City, State, and ZIP Code) 800 N. Quincy Street Arlington, Virginia 22217-5000		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Office of Naval Research		8b. OFFICE SYMBOL (If applicable)		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-89-J-1991	
8c. ADDRESS (City, State, and ZIP Code) 800 N. Quincy Street Arlington, Virginia 22217-5000			10. SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.
			WORK UNIT ACCESSION NO.		
11. TITLE (Include Security Classification) A Comparison of Several Theories which Use Jet Scaling to Predict Liftoff Heights of Turbulent-Jet Flames (Unclassified)					
12. PERSONAL AUTHOR(S) Jay A. Hammer					
13a. TYPE OF REPORT GALCIT Report FM 90-3		13b. TIME COVERED FROM 7/15/90 TO 7/15/91		14. DATE OF REPORT (Year, Month, Day) 1990 December 31	
				15. PAGE COUNT 24	
16. SUPPLEMENTARY NOTATION					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP			
			flame stability; burning fuel jets; flame models.		
19. ABSTRACT (Continue on reverse if necessary and identify by block number)					
<p>In this paper several previously developed theories for predicting liftoff heights of turbulent-jet flames are examined and compared. The five theories are: a turbulent flame speed theory, a cascade mixing time theory, a strain-rate theory, a scalar dissipation-rate theory and an empirical correlation. Each theory uses far-field scaling of isothermal, turbulent jets. Through a systematic algebraic development, the predictions of the theories are presented in a common form. They are then compared for their ability to predict the experimentally observed linear dependence of liftoff height on nozzle exit velocity. Following this, their ability to correctly predict the relative liftoff heights of different fuels is examined. Finally, the theories are extended to predict blowout and their predictions are compared to experimental results of several fuels.</p>					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. G. D. Roy/Dr. Spiro Lekoudis			22b. TELEPHONE (Include Area Code) (703) 696-4406		22c. OFFICE SYMBOL

FM 90-3

**A Comparison of Several Theories which Use Jet Scaling
to Predict Liftoff Heights of Turbulent-Jet Flames**

Jay A. Hammer



1. Title		2. Author	
3. Subject		4. Date	
5. Distribution		6. Availability	
7. Availability		8. Availability	
9. Availability		10. Availability	
11. Availability		12. Availability	
13. Availability		14. Availability	
15. Availability		16. Availability	
17. Availability		18. Availability	
19. Availability		20. Availability	
21. Availability		22. Availability	
23. Availability		24. Availability	
25. Availability		26. Availability	
27. Availability		28. Availability	
29. Availability		30. Availability	
31. Availability		32. Availability	
33. Availability		34. Availability	
35. Availability		36. Availability	
37. Availability		38. Availability	
39. Availability		40. Availability	
41. Availability		42. Availability	
43. Availability		44. Availability	
45. Availability		46. Availability	
47. Availability		48. Availability	
49. Availability		50. Availability	
51. Availability		52. Availability	
53. Availability		54. Availability	
55. Availability		56. Availability	
57. Availability		58. Availability	
59. Availability		60. Availability	
61. Availability		62. Availability	
63. Availability		64. Availability	
65. Availability		66. Availability	
67. Availability		68. Availability	
69. Availability		70. Availability	
71. Availability		72. Availability	
73. Availability		74. Availability	
75. Availability		76. Availability	
77. Availability		78. Availability	
79. Availability		80. Availability	
81. Availability		82. Availability	
83. Availability		84. Availability	
85. Availability		86. Availability	
87. Availability		88. Availability	
89. Availability		90. Availability	
91. Availability		92. Availability	
93. Availability		94. Availability	
95. Availability		96. Availability	
97. Availability		98. Availability	
99. Availability		100. Availability	

**Graduate Aeronautical Laboratories
California Institute of Technology
Pasadena, CA 91125**

December, 1990

Abstract

In this paper several previously developed theories for predicting liftoff heights of turbulent-jet flames are examined and compared. The five theories are: a turbulent flame speed theory, a cascade mixing time theory, a strain-rate theory, a scalar dissipation-rate theory and an empirical correlation. Each theory uses far-field scaling of isothermal, turbulent jets. Through a systematic algebraic development, the predictions of the theories are presented in a common form. They are then compared for their ability to predict the experimentally observed linear dependence of liftoff height on nozzle exit velocity. Following this, their ability to correctly predict the relative liftoff heights of different fuels is examined. Finally, the theories are extended to predict blowout and their predictions are compared to experimental results of several fuels.

List of Symbols

a	velocity gradient at extinction in a counterflow diffusion flame
c_i	constants, $i = 1, 2, 3, \dots$
c_ν	constant with dimensions of kinematic viscosity
C_p	specific heat at constant pressure
d	nozzle diameter
d^*	momentum diameter, $d(\rho_o/\rho_\infty)$
D	molecular diffusivity
$f_i(\eta)$	function of η as defined by a particular equation, $i = 1, 2, 3, \dots$
$F_n(\eta)$	"correction functions" to make theories match experimental results, $n = t, m, s, d$ and ϵ .
h	liftoff height
k	thermal conductivity
l	integral length scale
r	radial coordinate
S	maximum laminar flame speed
\bar{u}	mean velocity
u'	rms velocity
U_o	nozzle exit velocity
x	axial distance
Y	fuel mass fraction
Y_o	initial fuel mass fraction (balance is air)
η	r/x
κ	thermal diffusivity, $k/(\rho C_p)$
ν	kinematic viscosity
ρ	density
σ	strain rate

τ_c	fuel chemical time
τ_f	cascade mixing time
χ	scalar dissipation rate, $2D \nabla Y ^2$

Other symbols

$(.)_o$	nozzle exit conditions
$(.)_\infty$	reservoir conditions
$(.)_{st}$	stoichiometric conditions
$(.)_l$	conditions at maximum laminar flame speed
$(.)_{stab}$	assumed stabilization conditions, either $(.)_{st}$ or $(.)_l$
$(.)_{qu}$	quenching
$(.)_{cl}$	centerline
$(.)_b$	blowout
$(.)_t$	turbulent flame speed theory
$(.)_m$	cascade mixing time theory
$(.)_s$	strain-rate theory
$(.)_d$	scalar dissipation-rate theory
$(.)_e$	empirical correlation theory
$(.)_n$	name, representing t , m , s , d or e
$(\bar{})$	time average

1. Introduction

Several theories using far-field scaling of isothermal, turbulent jets have been developed to explain the dependence of the mean liftoff height above the nozzle exit, \bar{h} , on the flow parameters. By appropriately grouping the parameters in each theory, the dependence of \bar{h} on the nozzle velocity U_o , momentum diameter d^* , initial mass fraction of fuel Y_o and chemical and transport properties of the fuels is brought out.

The theories examined here are: the turbulent flame speed theory of Kalghatgi [1], the cascade mixing time theory of Broadwell et al. [2], the strain-rate theory of Miake-Lye and Hammer [3], the scalar dissipation-rate theory of Peters and Williams [4] and a theory developed by Pitts [5]. Each of the above theories is treated in a similar manner. First, through algebraic manipulation, an equation is derived involving \bar{h} , U_o , Y_o , fuel parameters, similarity functions of $\eta \equiv r/x$ for velocity, concentration, etc. and what will be called a correction function, $F_n(\eta)$, which is needed to make predictions match experimental results. (Ideally, $F_n(\eta) \equiv 1$ for each theory.) Next, the necessary shape of $F_n(\eta)$ to make the theory match results is found. Finally, setting $F_n(\eta) \equiv 1$, η is eliminated from the equation and the true predictions of the theory are given.

The remainder of the paper is organized as follows. First, experimental results of lifted flames are summarized and assumed scaling laws of isothermal jets are given. Next, each of the five theories is analyzed as described in the preceding paragraph. In the Comparisons section the similarities and differences of the theories are summarized.

2. Experimental results and assumed similarity profiles

2.1 Results for lifted jet flames

Kalghatgi [1] has found that for pure fuel issuing from a nozzle \bar{h} is proportional to U_o and independent of d for a given fuel, once \bar{h} reaches $\sim 10d$. In non-dimensional terms

this can be written as

$$\frac{\bar{h}}{d^*} = \tau'_c \frac{U_o}{d^*}, \quad (1)$$

where τ'_c is a chemical time dependent of the fuel. For small dilutions ($Y_o \approx 1$) $\bar{h}/U_o \propto (1/Y_o)$ [3] and so Eq. (1) can be written as

$$\frac{\bar{h}}{d^*} = \tau_c \frac{U_o}{d^*} \frac{Y_{st}}{Y_o}. \quad (2)$$

For larger dilutions \bar{h}/U_o was not found to be proportional to $1/Y_o$, although this may be due to near-field effects.

This liftoff behavior persists until the flame blows out at a maximum velocity U_b which is proportional to d , with the constant of proportionality depending on the fuel.

2.2 Assumed scaling laws of isothermal turbulent jets

The self-similar jet profiles which are used in the theories are the following:

$$\bar{u} = c_1 U_o \frac{d^*}{x} f_1(\eta) \quad (3)$$

$$f_1(\eta) = e^{-c'_1 \eta^2} \quad (3.1)$$

$$\bar{Y} = c_2 Y_o \frac{d^*}{x} f_2(\eta) \quad (4)$$

$$f_2(\eta) = e^{-c'_2 \eta^2} \quad (4.1)$$

$$u' = c_3 U_o \frac{d^*}{x} f_3(\eta) \quad (5)$$

$$f_3(\eta) = e^{-c'_3 \eta^2} \quad (5.1)$$

$$\sigma = c_4 U_o \frac{d^*}{x^2} f_4(\eta) \quad (6)$$

$$f_4(\eta) = e^{-c'_4 \eta^2} \quad (6.1)$$

$$\chi = c_5 \frac{U_o}{d^*} Y_o^2 \left(\frac{d^*}{x} \right)^4 f_5(\eta) \quad (7)$$

$$l = c_6 x \quad (8)$$

The above Gaussian functions (Eqs. (3.1), (4.1), (5.1) and (6.1)) are used only for estimating true predictions of the theories. Functions of any shape may be used in place of the Gaussian functions at the expense of greater complication in stating the final results.

3. Brief development of each of the five theories

In this section the five theories (turbulent flame speed, cascade mixing time, strain rate, scalar dissipation rate and the empirical theory) are examined. The descriptions do *not* exactly follow the authors' developments for each of the theories. This is done in order to bring out the similarities and differences in the theories. Major deviations from the authors' developments are noted.

3.1 Turbulent flame speed theory

The turbulent flame speed theory assumes that the mean velocity \bar{u} equals the turbulent flame speed u_t at the liftoff location specified by both axial x and radial r coordinates. Also, the liftoff location is assumed to lie along the contour of maximum laminar burning velocity, $\bar{Y} = Y_l$. u_t is assumed to depend on the flow properties as

$$\frac{u_t}{S} = c'_7(u'l/\nu_l)^{1/2} = c_7(u'x/\nu_l)^{1/2}. \quad (9)$$

There is some experimental support for this expression, although considerable scatter exists [6].

Kalghatgi [1] uses dimensional analysis in his development and eventually arrives at an expression for the turbulent flame speed. The current description starts with the assumption of the turbulent flame speed relation, Eq. (9), and arrives at an equation for \bar{h} . Its development is different from that of Kalghatgi in this way and in the use of the correction function described below.

Rather than the rigid requirement $\bar{u} = u_t$ at the liftoff position, in the present discussion we admit that jet turbulence is not homogeneous and isotropic and that there

will be differences at different radial locations η . The correction function for this theory is denoted as $F_t(\eta)$ and we require

$$\bar{u} = F_t(\eta)u_t, \quad (10)$$

where $F_t(\eta) \equiv 1$ for the turbulent flame speed theory to be exact. Squaring Eq. (10), using Eq. (9), and setting $x = h$ yields

$$\bar{u}^2 = S^2 c_7^2 \frac{u' \bar{h}}{\nu_l} F_t^2(\eta).$$

Using Eqs. (3) and (5),

$$c_1 U_o \frac{d^*}{\bar{h}} f_1(\eta) \frac{c_1}{c_3} \frac{f_1(\eta)}{f_3(\eta)} u' = S^2 c_7^2 \frac{u' \bar{h}}{\nu_l} F_t^2(\eta).$$

Using Eq. (4) and the assumption $\bar{Y} = Y_l$ to eliminate d^*/\bar{h} ,

$$\frac{c_1^2}{c_2 c_3} U_o \frac{Y_l}{Y_o} \frac{f_1^2(\eta)}{f_2(\eta) f_3(\eta)} = S^2 c_7^2 \frac{\bar{h}}{\nu_l} F_t^2(\eta).$$

Finally, solving for \bar{h} ,

$$\bar{h} = c_{t,1} \frac{Y_l}{Y_o} \frac{\nu_l}{S^2} U_o \frac{f_1^2(\eta)/(f_2(\eta) f_3(\eta))}{F_t^2(\eta)}, \quad (11)$$

where the constants have been absorbed into $c_{t,1}$.

Some important points to note from Eq. (11) are that at a fixed η , $\bar{h} \sim 1/Y_o$, $\bar{h} \sim U_o$, and \bar{h} does not depend on d^* . These relations will turn out to be true for *all* of the theories in this paper.

There are two remaining issues to address regarding the turbulent flame speed theory. The first is whether $f_t(\eta) \equiv \frac{f_1^2(\eta)}{f_2(\eta) f_3(\eta)} \approx \text{constant}$ as required for agreement with experimental liftoff results that there is a linear relation between \bar{h} and U_o for a given fuel. By determining the actual shape of $f_t(\eta)$ based on experimental measurements of $f_1(\eta)$, $f_2(\eta)$ and $f_3(\eta)$, the required correction function $F_t(\eta)$ can be determined. Alternatively,

once $f_t(\eta)$ is found, $F_t(\eta)$ can be set to 1 to show the true turbulent flame speed theory prediction without correction.

The second issue is whether the theory correctly predicts the relative liftoff heights of different fuels. This is measured by whether the factor $Y_l \nu_l / S^2$ in Eq. (11) agrees with experimental results of slopes of \bar{h} versus U_o lines for various fuels (τ'_c in Eq. (1)). Addressing this second issue is postponed until the Comparisons section where all five theories are compared.

Returning to the first issue, experimental data is used to approximate the functions, $f_1(\eta)$, $f_2(\eta)$ and $f_3(\eta)$ with Gaussian profiles as indicated in Eqs. (3.1), (4.1) and (5.1). For mean velocity, $c'_1 = 93.7$ from data of Chen and Rodi [7]. For fluctuating velocity, $c'_3 = 40.5$ is found from curve fitting data of Wygnanski and Fiedler [8] at $\bar{Y}/\bar{Y}_{cl} = 1/2$. This curve fit turns out to be about 30% high near the jet edge, $\eta = 0.2$, but will nevertheless be used here for simplicity. Finally, for concentration, $c'_2 = 57$ [7, 9]. This results in

$$f_t(\eta) = e^{-(2c'_1 - c'_2 - c'_3)\eta^2} = e^{-c'_t \eta^2} = e^{-90\eta^2}. \quad (12)$$

The ratio of $f_t^{1/2}$ at $\eta = 0.1$ and 0.2 (the approximate range of values encountered experimentally in lifted flames) is $e^{-45(.2^2 - .1^2)} = 0.26$. Therefore, $F_t(\eta)$ would need to be $\approx e^{-45\eta^2}$ and the ratio \bar{u}/u_t would need to vary by a factor of ~ 4 to make the turbulent flame speed theory match experimental results.

If $F_t(\eta)$ is set identically equal to 1, the predictions of a true turbulent flame speed theory can be found. Multiplying Eq. (11) by $Y_l/(Y_o d^*)$,

$$\frac{Y_l}{Y_o} \frac{\bar{h}}{d^*} = c_{t,1} \left(\frac{Y_l}{Y_o} \right)^2 \frac{\nu_l}{S^2} \frac{U_o}{d^*} f_t(\eta). \quad (13)$$

Solving for η from Eq. (4)

$$\frac{Y_l}{Y_o} \frac{\bar{h}}{d^*} = c_{t,1} \left(\frac{Y_l}{Y_o} \right)^2 \frac{\nu_l}{S^2} \frac{U_o}{d^*} f_t(f_2^{-1}(\frac{1}{c_2} \frac{Y_l}{Y_o} \frac{\bar{h}}{d^*})).$$

Using the Gaussian functions results in the power law dependence,

$$\frac{Y_l}{Y_o} \frac{\bar{h}}{d^*} = c_{t,2} \left(\left(\frac{Y_l}{Y_o} \right)^2 \frac{\nu_l}{S^2} \frac{U_o}{d^*} \right)^{1/(1-c'_1/c'_2)}. \quad (14)$$

The exponent $E_t \equiv \frac{1}{1-c'_1/c'_2} = -1.73$. Because $E_t < 0$, the turbulent flame speed theory actually predicts a decreasing liftoff height with increasing nozzle velocity for a fixed d^* and fuel.

3.2. Cascade mixing time theory

In a paper devoted primarily to blowout of flames, Broadwell et al. [2] proposed that lifted flames will be stabilized at a distance such that the mixing time, which is controlled by a turbulence cascade time, $\tau_f \sim x/u_{cl}$, is equal to a chemical time for the fuel, $\tau_{c,m}$. Their theory is one dimensional because τ_f is assumed to have no dependence on η . $\tau_{c,m}$ is assumed to be proportional to κ/S^2 in accordance with simple one-dimensional laminar flame speed theories (e.g., Glassman [10]). A discussion of what value of κ should be used is postponed until the Comparisons section.

For the present description, a correction function $F_m(\eta)$ will be added to the theory as was done with the turbulent flame speed theory. Using Eq. (3) the liftoff requirement becomes

$$\frac{\bar{h}^2}{U_o d^*} F_m(\eta) = c_{10} \frac{\kappa}{S^2}. \quad (15)$$

Assuming the flame stabilizes along the mean stoichiometric contour (an assumption that was *not* necessary in the original one-dimensional formulation) and using Eq. (4),

$$\bar{h} = c_{m,1} \frac{Y_{st}}{Y_o} \frac{\kappa}{S^2} U_o \frac{1/f_2(\eta)}{F_m(\eta)}. \quad (16)$$

As with the turbulent flame speed theory, at constant η , $\bar{h} \sim U_o$, $\bar{h} \sim 1/Y_o$ and \bar{h} is independent of d^* .

To find the true predicted liftoff height, $F_m(\eta)$ is set to 1 in Eq. (16) to get

$$\frac{Y_{st}}{Y_o} \frac{\bar{h}}{d^*} = c_{m,1} \left(\frac{Y_{st}}{Y_o} \right)^2 \frac{\kappa}{S^2} \frac{U_o}{d^*} 1/f_2(\eta). \quad (17)$$

Noting the similarity to Eq. (13), we can write

$$\frac{Y_{st}}{Y_o} \frac{\bar{h}}{d^*} = c_{m,2} \left(\left(\frac{Y_{st}}{Y_o} \right)^2 \frac{\kappa}{S^2} \frac{U_o}{d^*} \right)^{1/2}. \quad (18)$$

The Gaussian shape of $f_2(\eta)$ was not used in deriving Eq. (18) because no other functions were involved, and, the exponent, $E_m = 1/2$, is exact. In fact, Eq. (18) can easily be derived from the strictly one-dimensional theory.

3.3 Strain-rate theory

A theory very similar to the above mixing time theory is the maximum strain-rate theory of Miake-Lye and Hammer [3]. Here, however, the liftoff condition is that the large scale strain rate σ cannot exceed a maximum strain rate $\sigma_{crit} \sim S^2/\kappa$. Also, σ is allowed to have a radial dependence, hence making this a two-dimensional theory. The flame is assumed to lie along a contour on which stoichiometric conditions are most probable. To approximate this condition, it is assumed that $\bar{Y} = Y_{st}$.

Using a correction function $F_s(\eta)$ the liftoff requirement becomes

$$\sigma = c_4 U_o \frac{d^*}{x^2} f_4(\eta) = c_{11} \frac{S^2}{\kappa} F_s(\eta).$$

Solving for \bar{h} ,

$$\bar{h} = c_{s,1} \frac{Y_{st}}{Y_o} \frac{\kappa}{S^2} U_o \frac{f_4(\eta)/f_2(\eta)}{F_s(\eta)}, \quad (19)$$

which is very similar to Eq. (16). The only difference is that the radial strain function $f_4(\eta)$ appears. Therefore, to match experimental liftoff results, $F_s(\eta) = f_4(\eta)/f_2(\eta)$.

Unfortunately, $f_4(\eta)$ has not been experimentally measured. Nor was it completely defined in Ref. [3] where the authors described it as the large-scale strain rate between

large vortical structures. One possibility is to use $\frac{\partial \bar{u}}{\partial r}$ for σ . This, however, will not give a Gaussian profile. Instead, in the present paper, a Gaussian profile is used which matches $\frac{\partial \bar{u}}{\partial r}$ at $\eta = 0.1$ and $\eta = 0.2$ to get

$$c'_4 = c'_1 - \log(.2/.1)/(.2^2 - .1^2) = 71.$$

This makes $f_s(\eta) \equiv f_4(\eta)/f_2(\eta) = e^{-c'_s \eta^2}$ where $c'_s = c'_4 - c'_2 = 14$.

The true predictions of the theory are then

$$\frac{Y_{st}}{Y_o} \frac{\bar{h}}{d^*} = c_{s,2} \left(\left(\frac{Y_{st}}{Y_o} \right)^2 \frac{\kappa}{S^2} \frac{U_o}{d^*} \right)^{1/(1-c'_s/c'_2)}. \quad (20)$$

The exponent $E_s \equiv 1/(1 - c'_s/c'_2) = 1.3$, although, due to the approximation in determining c'_4 , this number should be treated with some skepticism.

3.4 Scalar dissipation-rate theory

Peters and Williams [4] have proposed that the liftoff height is determined by extinction of thin flamelets. Through an asymptotic analysis they find that the scalar dissipation rate χ is the parameter which determines whether a flamelet will be quenched. The criterion for flame stabilization is that the probability of $\chi < \chi_{qu}$ at stoichiometric conditions is greater than some minimum probability, where χ_{qu} is a limiting scalar dissipation rate for the particular fuel. Using percolation theory, this requirement is simplified to $\bar{\chi}_{st} < \chi_{qu}$. In the present formulation, we will again add a correction function $F_d(\eta)$ and require

$$\bar{\chi}_{st} = \chi_{qu} F_d(\eta). \quad (21)$$

Here, $\bar{\chi}_{st}$ is the average scalar dissipation rate conditional on $Y = Y_{st}$. In order to make use of experimental measurements in jets, $\bar{\chi}_{st}$ must be related to the unconditional average dissipation rate $\bar{\chi}$. For this we assume that $\bar{\chi}_{st} = \bar{\chi} f_7(Y_{st}/\bar{Y}_{cl}, \eta)$, and that $\bar{Y} = Y_{st}$. Then $\bar{\chi}_{st} = \bar{\chi} f_7(f_2(\eta), \eta)$. Combining this with Eq. (21),

$$\bar{\chi} = \bar{\chi}_{st} / \tilde{f}_7(\eta) = \chi_{qu} F_d(\eta) / \tilde{f}_7(\eta).$$

Now, assuming $\tilde{f}_7(\eta)$ is not a strong function of η (or equivalently absorbing it into $F_d(\eta)$) we arrive at the condition

$$\bar{\chi} = c_{12} \chi_{qu} F_d(\eta) \quad (22)$$

for determining the liftoff height.

After that somewhat complicated introduction, we are now ready to proceed using Eqs. (22), (7) and (4) as with the previous theories to get

$$\bar{h} = c_{d,1} \frac{Y_{st}}{Y_o} \frac{Y_{st}^2}{\chi_{qu}} U_o \frac{f_5(\eta)/f_2^3(\eta)}{F_d(\eta)}. \quad (23)$$

Like the previous theories at constant η , $\bar{h} \sim U_o$, $\bar{h} \sim 1/Y_o$ and \bar{h} is independent of d^* . Unlike the previous theories an additional Y_{st}^2 appears in the result, but Y_{st}^2/χ_{qu} is simply a chemical time relating the relative liftoff heights of different fuels.

For estimating the shape of $f_5(\eta)$, results of Dowling's [9] point measurements of concentration are used. Using Taylor's hypothesis along with the mean off-centerline velocity, he states that "the mean value of [the estimated scalar dissipation rate] is almost independent of $[\eta]$...". Therefore, $f_5(\eta)$ is approximated as a constant, and $F_d(\eta)$ would need to be equal to $f_d(\eta) \equiv f_5(\eta)/f_2^3(\eta) \approx e^{-171\eta^2}$ for the scalar dissipation-rate theory to match experimental results. Peters and Williams approximated $\bar{\chi}$ as $U_o d^* (\frac{\partial \bar{Y}}{\partial r})^2$ which makes $f_5(\eta) \sim \eta^2 e^{-2c'_2 \eta^2}$ if the Gaussian profile for $f_2(\eta)$ given by Eq. (4.1) is used. The current discussion will assume that $f_5(\eta) \approx \text{constant}$, however.

Setting $F_d(\eta) \equiv 1$ in Eq. (23) results in

$$\frac{Y_{st}}{Y_o} \frac{\bar{h}}{d^*} = c_{d,2} \left(\left(\frac{Y_{st}}{Y_o} \right)^2 \frac{Y_{st}^2}{\chi_{qu}} \frac{U_o}{d^*} \right)^{1/(1-c'_d/c'_2)}, \quad (24)$$

where $c'_d = -3c'_2$ and therefore $E_d \equiv 1/(1 - c'_d/c'_2) = 1/4$.

It should be pointed out that Peters and Williams have not stated that their theory should be applicable to diluted fuels where $Y_o < 1$. The idea that the quenching scalar dissipation rate χ_{qu} is a constant is based in part on the assumption of boundary

conditions of the flamelets being pure fuel on one side and pure oxidizer on the other. However, for comparison with other theories in this paper, the effect of Y_o has been included with the assumption that χ_{qu} would not change.

One other point to be made concerns the constant c_5 in Eq. (7). Although Eq. (7) is expected to be correct in the limit $Re \rightarrow \infty$, at Reynolds numbers of 5000 and 16000, Dowling finds $c_5 = 54$ and 32 respectively. This would cause even further disagreement between experimental results and the scalar dissipation-rate theory in this range of Reynolds numbers.

3.5 Empirical correlation proposed by Pitts

Pitts [5] has proposed a correlation for predicting liftoff heights which is based on jet scaling (with some caveats). He proposes that flames stabilize along the contour of maximum laminar burning velocity ($\bar{Y} = Y_l$) at a location where

$$\bar{u} = \frac{1}{c_\nu} \frac{x}{d^*} x S^2 Y_l^2. \quad (25)$$

The first difficulty with the correlation is that c_ν is a constant with dimensions of kinematic viscosity, independent of fuel and oxidizer, which cannot physically be correct. A possible fix to this would be to replace c_ν with ν_l , although this would change predictions slightly for different fuels.

The second difficulty concerns the factor Y_l^2 in Eq. (25). Whether this should be replaced by $(Y_l/Y_o)^2$ or $(Y_l/Y_o)Y_l$ or left as Y_l^2 when $Y_o < 1$ is not stated by Pitts. The correlation was developed to match data for pure fuels only, and so this was not a consideration. In the present discussion Y_l^2 will be replaced by $(Y_l/Y_o)Y_l$ because the other two choices result in \bar{u}_{stab} depending on x/d^* , or, through simple algebraic manipulation, on Y_o . It is assumed in jet far-field scaling that the flow is independent of these parameters.

Replacing Y_l^2 with $Y_l(Y_l/Y_o)$ in Eq. (25) and adding a correction function $F_e(\eta)$,

$$\bar{u} = \frac{1}{c_\nu} \frac{x}{d^*} x S^2 Y_l(Y_l/Y_o) F_e(\eta). \quad (26)$$

Using Eq. (4) to eliminate $(x/d^*)(Y_l/Y_o)$ better illustrates the assumed dependence of \bar{u} on the flow variables:

$$\bar{u}/S = c_2 Y_l(xS/c_\nu) f_2(\eta) F_e(\eta). \quad (26.1)$$

It is difficult to justify theoretically this relationship, especially with the factor Y_l , but at least there is no remaining x/d^* which would put it out of the realm of jet scaling theories.

Using Eq. (3) in Eq. (26),

$$\bar{h} = c_{e,1} \frac{Y_l}{Y_o} \frac{c_\nu}{Y_l S^2} U_o \frac{f_1(\eta)/f_2^2(\eta)}{F_e(\eta)}, \quad (27)$$

where Y_l has been explicitly left in the numerator and denominator. For a linear relation between U_o and \bar{h} , $F_e(\eta)$ must equal $f_e(\eta) \equiv f_1(\eta)/f_2^2(\eta) = e^{-c'_e \eta^2}$, where $c'_e = c'_1 - 2c'_2 = -20$. Pitts effectively used such a correction function by using a slightly higher value of c'_1 than had been found in turbulent jet experiments.

The true predictions of the theory are

$$\frac{Y_l}{Y_o} \frac{\bar{h}}{d^*} = c_{e,2} \left(\left(\frac{Y_l}{Y_o} \right)^2 \frac{c_\nu}{Y_l S^2} \frac{U_o}{d^*} \right)^{E_e}, \quad (28)$$

where $E_e = 1/(1 - c'_e/c'_2) = 0.74$.

4. Comparison of theories

Each of the theories investigated produces a prediction of the form

$$\bar{h} = c_{n,1} (Y_{stab}/Y_o) \tau_{c,n} U_o f_n(\eta), \quad (29)$$

where Y_{stab} is the assumed stabilization concentration, Y_{st} for the mixing time, strain-rate and scalar dissipation-rate theories and Y_l for the turbulent flame speed and empirical

theories. $\tau_{c,n}$ is a fuel-dependent chemical time and $f_n(\eta)$ is a function which depends on jet similarity functions. The above relation is a direct result of the fact that the proposed stabilization conditions of each of the theories can be expressed as

$$\frac{\bar{h}}{\bar{u}_{cl}} \sim \tau_{c,n} f_{n,2}(\eta),$$

$$\bar{Y} = Y_{stab},$$

where $f_{n,2}(\eta) = f_n(\eta)f_2(\eta)$. By eliminating η from Eq. (29) using Eq. (4), the liftoff prediction can be stated as

$$\frac{Y_{stab}}{Y_o} \frac{\bar{h}}{d^*} = f_{n,3} \left(\left(\frac{Y_{stab}}{Y_o} \right)^2 \tau_{c,n} \frac{U_o}{d^*} \right). \quad (30.1)$$

Approximating $f_n(\eta)$ and $f_2(\eta)$ with Gaussian functions results in a simple power law form for the function $f_{n,3}$

$$\frac{Y_{stab}}{Y_o} \frac{\bar{h}}{d^*} = c_{n,2} \left(\left(\frac{Y_{stab}}{Y_o} \right)^2 \tau_{c,n} \frac{U_o}{d^*} \right)^{E_n}. \quad (30.2)$$

This section is organized into four subsections. First, predictions for a fixed fuel (and fixed chemical and transport parameters) are compared by examining $f_n(\eta)$. Second, the necessary parameters for comparing different fuels via different expressions for $Y_{stab}\tau_{c,n}$ are tabulated. The third subsection uses these parameters to compare experimental results for liftoff heights with predictions for *different* fuels. The last subsection addresses blowout predictions.

4.1 Predictions for a single fuel

Table 1 lists the following properties of each theory: expression for $f_n(\eta)$ in terms of similarity functions f_1 , f_2 , etc.; the approximate value of the c'_n when $f_n(\eta)$ is approximated as $e^{-c'_n \eta^2}$; the the exponent in Eq. (30.2), $E_n \equiv 1/(1 - c'_n/c'_2)$; and the expression for $Y_{stab}\tau_{c,n}$.

Of the above theories, only the turbulent flame speed theory predicts a decreasing \bar{h} with increasing U_o , as evidenced by $E_n < 0$. The scalar dissipation-rate theory has the largest, negative c'_n and consequently predicts a dependence of $U_o^{0.25}$. Other theories have a more nearly linear dependence in agreement with experimental results. Due to lack of data on the strain-rate profile, the results for the strain-rate theory are less certain than those of the other theories.

Table 1
Summary of Theory Predictions

Theory	$f_n(\eta)$	c'_n	E_n	$Y_{stab}\tau_{c,n}$
turbulent flame speed	$f_1^2/(f_2 f_3)$	90	-1.73	$Y_l(\nu_l/S^2)$
cascade mixing time	$1/f_2$	-57	1/2	$Y_{st}(\kappa/S^2)$
strain rate	f_4/f_2	14	1.3	$Y_{st}(\kappa/S^2)$
scalar dissipation	f_5/f_2^3	-171	0.25	$Y_{st}(Y_{st}^2/\chi_{qu})$
empirical correlation	f_1/f_2^2	-20	0.74	$Y_l(c_\nu/(Y_l S^2))$

4.2 Properties for evaluation of chemical times

In Table 2, values for Y_{st} , Y_l and S are taken from Kalghatgi [1]. In accordance with turbulent flame speed correlations, ν_l is evaluated at the unburned condition, assumed to be 300 K.

Evaluation of κ is somewhat more difficult. Broadwell et al. used for a value for κ that of air at 2000K for all the fuels because the hydrocarbons have similar flame temperatures. (Hydrogen's temperature is somewhat higher.) Miake-Lye and Hammer used κ of stoichiometric unburned mixtures at 2200 K, although this required considerable extrapolation for the thermal conductivities of some of the components. More detailed inspection of the laminar flame speed theories reveals that ρ should be evaluated at the cold, unburned condition. The theory of Mallard and Le Chatelier [10] requires

C_p to be evaluated at a temperature between the flame temperature and the unburned temperature, and k to be evaluated near the flame temperature for the most physical interpretation of the theory. The more complex theory of Zeldovich, Frank-Kamenetskii and Semenov [10] requires that C_p and k be constant. Therefore, in the Table 2, ρ and C_p are evaluated at stoichiometric, unburned conditions, and k is evaluated at the adiabatic flame temperature. Because reliable data for k of the products, H_2O and CO_2 , could not be found, k for N_2 at the flame temperature is used.

Values of χ_{qu} are obtained from the equation of Peters and Williams

$$\chi_{qu} = \chi_{st} \approx 4aY_{st}^2[erfc^{-1}(2Y_{st})]^2, \quad (31)$$

where the velocity gradient at extinction in a counterflow diffusion flame a is taken from the data of Tsuji and Yamaoka [11] for CH_4 and C_3H_8 and from the computations of Dixon-Lewis et al. [12] for H_2 . No value has been found for C_2H_4 .

Also included in Table 2 are experimental results for the slopes of the \bar{h} versus U_o lines from the data of Kalghatgi [1]. The final column gives data on blowout in the form of d^*/U_b taken from Broadwell et al. who used the data of Kalghatgi [13]. These last two columns will be compared with predictions in sections 4.3 and 4.4.

Table 2

Fuel properties and experimental results for liftoff and blowout

Fuel	Y_{st}	Y_l	S	ν_l	κ_{st}	χ_{qu}	\bar{h}/U_o	d^*/U_b
			m/s	$10^{-6}m^2/s$	$10^{-6}m^2/s$	s^{-1}	ms	μs
CH_4	0.0549	0.058	0.39	15.9	106	4.9	2.42	46.6
C_2H_4	0.0632	0.073	0.75	15.1	106	—	0.672	19.4
C_3H_8	0.060	0.068	0.45	14.6	102	6.1	1.91	51.2
H_2	0.0284	0.076	3.06	30.1	113	76.1	0.030	0.178

4.3 Comparison of predicted and measured liftoff heights of different fuels

Several points can be noted about the expressions for $Y_{stab}\tau_{c,n}$ given in Table 1. All contain a $1/S^2$ term, except the scalar dissipation-rate theory. The expressions for the mixing time and strain-rate theories are identical. Also the empirical correlation's expression would agree with that of the turbulent flame speed theory if c_ν/Y_l were replaced with ν_l in Eq. (26.1).

Using the properties in Table 2, the relative predicted liftoff slopes ($Y_{stab}\tau_{c,n}$) of the fuels are evaluated and tabulated in Table 3. For ease of comparison, the values have been normalized by the value for methane, as have the experimental results for \bar{h}/U_o . In addition, the deviation of the prediction from the experimental result is listed in percent. All of the theories correctly predict the relative liftoff heights of different fuels to within $\pm \sim 30\%$ except for the turbulent flame speed's prediction for hydrogen.

Table 3

Comparison of measured and predicted relative liftoff heights for several fuels

Fuel	\bar{h}/U_o	turb. f.s.	mix. time	strain rate	scalar diss.	empirical
CH_4	1	1	1	1	1	1
C_2H_4	0.278	0.323 (+16%)	0.311 (+12%)	0.311 (+12%)	—	0.270 (-3%)
C_3H_8	0.789	0.810 (+3%)	0.788 (+0%)	0.788 (+0%)	1.05 (+33%)	0.749 (-5%)
H_2	0.0124	0.0404 (+226%)	0.00896 (-28%)	0.00896 (-28%)	0.00891 (-28%)	0.0162 (+31%)

4.4 Blowout predictions and experimental results

By assuming that blowout occurs when the flame base reaches some minimum value of η , η_b , each of the theories can be extended to predict a blowout velocity U_b . This

assumption is equivalent to the criterion that h_b be proportional to the axial distance at which $\bar{Y}_{cl} = \bar{Y}_{stab}$, where $\bar{Y}_{stab} = \bar{Y}_{st}$ for the mixing, strain-rate and scalar dissipation-rate theories, and $\bar{Y}_{stab} = \bar{Y}_l$ for the turbulent flame speed and empirical theories.

Using Eq. (4) to eliminate \bar{h} from Eq. (29),

$$\frac{d^*}{\bar{U}_b} = (c_{n,1}/c_2) \left(\frac{\bar{Y}_{stab}}{\bar{Y}_o} \right)^2 \tau_{c,n} f_n(\eta_b) / f_2(\eta_b). \quad (32)$$

Taking η_b to be the same for all fuels results in the blowout prediction

$$\frac{d^*}{\bar{U}_b} = c_{n,3} \left(\frac{\bar{Y}_{stab}}{\bar{Y}_o} \right)^2 \tau_{c,n}. \quad (33)$$

Values of d^*/\bar{U}_b can be compared with $\bar{Y}_{stab}^2 \tau_{c,n}$ for each theory. These are shown in Table 4, where the values have again been normalized by those of CH_4 .

Only the authors of the mixing time theory and the strain-rate theory have actually proposed this scheme for predicting blowout. Pitts, in his empirical correlation development, began by assuming that blowout occurs when the stabilization contour reaches its maximum radius r_{max} ; this is equivalent to choosing a particular value of η_b . He then, however, found an empirical expression for the local velocity at blowout which is inconsistent with Eqs. (25) and (26). Effectively, he eliminated \bar{Y}_l from Eq. (26.1) in his discussion of blowout which then changed $\tau_{c,e}$ to c_ν/S^2 . In the current discussion, the expression of $\tau_{c,e}$ consistent with Eq. (26.1) is used.

All of the theories make reasonable predictions about blowout except in the case of hydrogen where the turbulent flame speed and empirical theories are far off. This is due to these theories using the assumption that $\bar{Y}_{stab} = \bar{Y}_l$ rather than \bar{Y}_{st} as the remaining theories assume. For hydrogen, \bar{Y}_l is very different from \bar{Y}_{st} , and the choice will affect predictions greatly.

Table 4

Comparison of blowout predictions and experimental results

Fuel	d^*/U_b	turb. f.s.	mix. time	strain	scalar diss.	empirical
CH_4	1	1	1	1	1	1
C_2H_4	0.416	0.407 (-2%)	0.358 (-14%)	0.358 (-14%)	—	0.341 (-18%)
C_3H_8	1.10	0.950 (-14%)	0.861 (-22%)	0.861 (-22%)	1.15 (+5%)	0.879 (-20%)
H_2	0.00382	0.0529 (+1300%)	0.00464 (+21%)	0.00464 (+21%)	0.00460 (+20%)	0.0213 (+460%)

5. Conclusions

Five theories predicting turbulent jet flame liftoff, each utilizing self-similar properties of isothermal jets, have been investigated. The theories are: a turbulent flame speed theory, a cascade mixing time theory, a maximum strain-rate theory, a limiting scalar dissipation-rate theory and an empirical theory. Each predicts a relation between \bar{h} and flow variables of the form

$$\bar{h} = c_{n,1}(Y_{stab}/Y_o)\tau_{c,n}U_o f_n(\eta), \quad (29)$$

with different expressions for Y_{stab} , $\tau_{c,n}$ and $f_n(\eta)$. By using the equation for the mean concentration field and by expressing the similarity functions as Gaussian, η was eliminated from Eq. (29) to arrive at the relation

$$\frac{Y_{stab}}{Y_o} \frac{\bar{h}}{d^*} = c_{n,2} \left(\left(\frac{Y_{stab}}{Y_o} \right)^2 \tau_{c,n} \frac{U_o}{d^*} \right)^{E_n}. \quad (30.2)$$

(The one-dimensional cascade mixing time theory did not require the assumption of Gaussian functions to arrive at Eq. (30.2).)

The five theories were compared for their ability to correctly predict the experimentally observed linear dependence of \bar{h} on U_o by examining the exponent E_n . They were also compared for their ability to predict the relative liftoff heights of *different* fuels by examining the expression for $Y_{stab}\tau_{c,n}$. Finally, the theories were extended to predict blowout and compared to experimental results.

Because the theories have much in common it is difficult to conclude which one is the best. The turbulent flame speed, cascade mixing time and scalar dissipation-rate theories all predict highly nonlinear relations between \bar{h} and U_o . The strain-rate field in turbulent jets has not been measured and as a result the strain-rate theory cannot be properly evaluated. The empirical correlation has no physical basis. The predictions of relative liftoff heights of different fuels were all within $\pm 30\%$ for all of the theories and all of the fuels except the turbulent flame speed's prediction for hydrogen. Blowout predictions of all the theories are within $\pm 25\%$ except for the turbulent flame speed and empirical theories' predictions for hydrogen.

The similarities in the theories illustrate their limitations in understanding lifted diffusion flames. To make significant progress in understanding the liftoff mechanism, more detailed physical models need to be developed.

6. Acknowledgement

This work was supported by the Office of Naval Research under contract number N00014-89-J-1991.

References

1. Kalghatgi, G. T. "Liftoff Heights and Visible Lengths of Vertical Turbulent Jet Diffusion Flames in Still Air," *Combustion Science and Technology*, **41**, (1984), 17-29.
2. Broadwell, J. E., Dahm, W. J. A., and Mungal, M. G. "Blowout of Turbulent Diffusion Flames," *Twentieth Symposium (International) on Combustion*, 1985, 303-310.
3. Miake-Lye, R. C. and Hammer, J. A. "Lifted Turbulent jet Flames: A Stability Criterion Based on the Jet Large-Scale Structure," *Twenty-Second International Symposium on Combustion*, 1989, 817-824.
4. Peters, N. and Williams, F. A. "Liftoff Characteristics of Turbulent Jet Diffusion flames," *AIAA J.*, **21(3)**, March, 1983, 423-429.
5. Pitts, W. M. "Importance of Isothermal Mixing Processes to the Understanding of Lift-Off and Blowout of Turbulent Jet Diffusion Flames," *Combustion and Flame*, **76**, (1989), 197-212.
6. Andrews, G. E., Bradley, D. and Lwakamamba, S. B. "Measurement of Turbulent Burning Velocity for Large Turbulent Reynolds Numbers," *Sixteenth Symposium (International) on Combustion*, 1977, 655-663.
7. Chen, C. J. and Rodi, W. "Vertical Turbulent Buoyant Jets," Pergamon, 1980.
8. Wygnanski, I. and Fielder, H. E. "Some Measurements in the Self-Preserving Jet," *J. Fluid Mech.*, **38**, (1969), 577-612.
9. Dowling, D. R. "Mixing in Gas Phase Turbulent Jets," Ph.D. Thesis, Caltech, 1988.

10. Glassman, I. **Combustion**, Academic Press, Inc., New York, 1977, pp. 114 - 125.
11. Tsuji, H. and Yamaoka, F., "Structure of Counterflow Diffusion Flames in the Forward Stagnation Region of a Porous Cylinder," Twelfth Symposium (International) on Combustion, (1969), 997-1005.
12. Dixon-Lewis, G., David, T., Gaskell, P. H., abstract for "Structure and Properties of Methane-Air and Hydrogen-Air Counterflow Diffusion Flames," Archivum Combustionis, **6** (1), 3-21.
13. Kalghatgi, G. T. "Blowout Stability of Gaseous Jet Diffusion Flames. Part I: In Still Air," Combustion Science and Technology, **26**, (1981), 233-239.